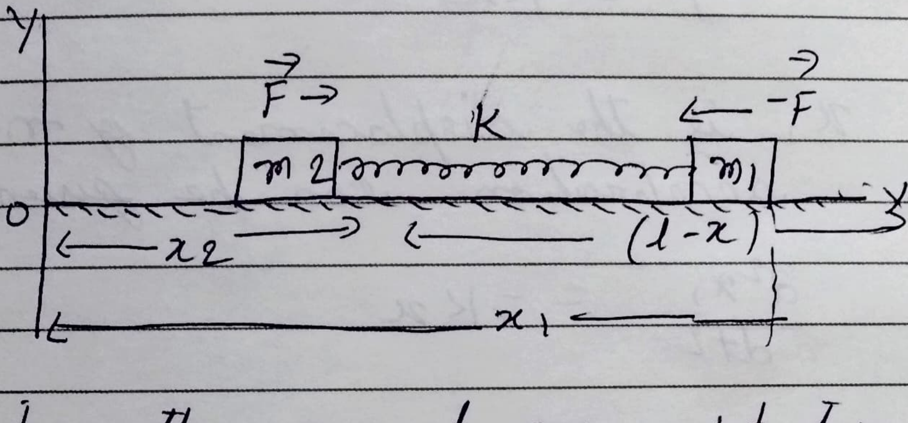
Two body oscillation →

A system of two bodies connected by a spring so that both are free to oscillate simple harmonically along the length of the spring constitutes a two-body Harmonic oscillator or coupled oscillator.

e.g — Diatomic molecule like CO, HCl etc



consider the general case, let two masses m_1 and m_2 connected by a horizontal light (mass less) spring of force constant K, so that they oscillate simple harmonically along the x axis.

Let the equilibrium length be l
If x_1 and x_2 are the coordinates of the ends of the spring and let 'x' be the extention of the spring when it oscillates then

$$x = (x_1 - x_2) - l$$

or $$(x_1 - x_2) = x \pm l$$

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If spring is stretched, x is +ve

we are assuming it to be positive

from fig ;
the force (F) exerted by the spring on m_1 and m_2 is equal on magnitude but opposite in sign.

The magnitude of each forces are

$$F = Kx$$

If x_1 is the displacement of m_1, then the acceleration can be given by

$$\frac{d^2x_1}{dt^2} = -Kx$$

$$\text{or} \quad m_1\frac{d^2x_1}{dt^2} = -Kx \quad \text{——} (1)$$

If x_2 is the displacement for m_2 then

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acceleration can be given by

$$\frac{d^2x_2}{dt^2} = Kx$$

$$\text{or} \quad m_2\frac{d^2x_2}{dt^2} = Kx \quad \text{——} (2)$$

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We have

$$m_1 \frac{d^2x_1}{dt^2} = -Kx \text{ and } \quad \text{—— (1)}$$

$$m_2 \frac{d^2x_2}{dt^2} = Kx \quad \text{—— (2)}$$

Differential from of SHM for the two masses m_1 and m_2

Multiplying eqn (1) by m_2 and eqn (2) by m_1 and subtracting, we get

$$m_2 m_1 \frac{d^2x_1}{dt^2} = -Kx m_2$$

and $m_1 m_2 \frac{d^2x_2}{dt^2} = Kx m_1$

$$m_1 m_2 \frac{d^2x_1}{dt^2} - m_1 m_2 \frac{d^2x_2}{dt^2} = -m_2 Kx - m_1 Kx$$

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$$m_1 m_2 \frac{d^2}{dt^2}(x_1 - x_2) = -Kx(m_1 + m_2)$$

$$\frac{m_1 m_2}{(m_1 + m_2)} \frac{d^2}{dt^2}(x_1 - x_2) = -Kx$$

l is a constant so,

$$\therefore \frac{m_1 m_2}{(m_1 + m_2)} \frac{d^2}{dt^2}(x) = -Kx$$

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$$\mu \frac{d^2}{dt^2}(x) = -Kx \quad \text{--- (3)}$$

where $\mu \rightarrow$ reduced mass $= \frac{m_1 m_2}{(m_1 + m_2)}$

of the system

from (3)

$$\frac{d^2x}{dt^2} = \frac{-K}{\mu}x$$

or $$\frac{d^2x}{dt^2} + \frac{K}{\mu}x = 0 \quad \text{--- (4)}$$

This is identical relation as oscillating body

Here, μ is the reduced
x is the relative

$$\frac{d^2x}{dt^2} + \omega^2 x = 0 \quad \left\{\omega^2 = \frac{K}{\mu}\right\}$$

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Thus the system oscillates simple harmonically with a time period

$\omega = \sqrt{\frac{K}{\mu}}$

$$T = \frac{2\pi}{\omega}$$

$$\boxed{\text{or } T = \frac{2\pi}{\sqrt{\frac{K}{\mu}}} = 2\pi\sqrt{\frac{\mu}{K}}}$$

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and frequency

$$\nu = n = \frac{1}{T}$$

$$\boxed{\nu = \frac{1}{2\pi}\sqrt{\frac{K}{\mu}}}$$

Two body system oscillate with same time period and frequency as a one body oscillator.

— x —